Dynamic Localization in Quantum Wires

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Abstract

In the paper the dynamic localization of charged particle (electron) in a quantum wire under the external non-uniform time-dependent electric field is considered. The electrons are trapped in a deep 'dynamic' quantum wells which are the result of specific features of the potential imposed on 2D electron gas: the scale of spatial nonuniformity is much smaller then the electron mean free path $(L_1 \ll \bar{l})$ and the frequency is much greater then τ^{-1}, where τ is the electron free flight time. As a result, the effect of this field on the charged particle is in a sense equivalent to the effect of a time-independent effective potential, that is a sequence of deep 'dynamic' quantum wells were the elelctrons are confined. The possible consequeces of this effect are also discussed and similarity with the classical Paul traps are emphasized.

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1 Introduction

To the author's knowledge, the term 'dynamic localisation' was coined by D.H.Dunlop and V.M.Kenkre in their well-known paper [1], where they considered charged particle motion on a liner chain of sites m $(-\infty < m < \infty)$ under the combined action of a time-dependent electric field $E(t) = E_0 \sin \omega t$ in the direction of the lattice and of nearest-neghbor coupling V. The Hamiltonian considered in [1] is of the form:

$$H(t) = V \sum_{-\infty}^{\infty} (\mid m >< m+1 \mid + \mid m+1 >< m \mid) - eE(t)d \sum_{-\infty}^{\infty} m \mid m >< m \mid,$$

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where $\mid m >$ represents Wannier state localised on the lattice site m, d is the lattice constatnt, V is the nearest-neigbbor intersite overlap integral and e is the particle charge. One of the main results of the paper [1] is that under certain conditions charged particle has to be localised in such a lattice, namely, the mean-square displacement $< m >$ is to be bounded if the ratio of the field magnitude to the field frequency is a root of J_0, where $J_n, (n = 0, \pm 1, \pm 2, ...)$ is the ordinary Bessel functions in terms of which the correspoding exponentials are expanded.

On the other hand, there is very similar but somewhat different type of charged particle localisation which also could be called 'dynamic'. It is the charged particle confinement in a nonuniform rapidly oscillating field which for the first time was considered in the framework of classical physics by W. Paul and M. Raether [2]. This seminal work was the starting point for the development of something which was called 'Paul traps' and which later became one of the main tools for trapping a few and even single ions for the purposes of ions cooling and high-resolution spectroscopy.

The point is that, as is known from calssical electrodynamics, there are no absolute maxima and minima of a potential in the electric field free of charges. As a consequence, the localisation of charged particles in this field is impossible if one means under localisation such a state where the particle with energy less than some definite value cannot leave the bound region under any initial conditions (Earenshow Theorem). However, as it was shown by W. Paul and M. Raether [2], the localisation of a particle in a *non-uniform high-frequency* electromagnetic field is possible. It turns out that under definite conditions the force acting on a particle can be represented by means of some effective time-independent potential. The force also does not depend on the sign of the particle charge and the particle can be localised in the space region where effective potential has its minimum.

It is quite clear that consistent description of ions trapping and their subsequent cooling in Paul traps needs to be treated in the quantum mechanical framework. Such quantum-mechnical analysis of the particle motion in a rapidly oscillating field has been done by R. J. Cook, D.G. Shankland and A.L. Wells [3]. Having supposed the amplitude of the particle wave function slowly varying function of time (to compare with short time interval $2\pi/\omega$, where ω is the field frequency), and making average over time much greater than $2\pi/\omega$, the authors of [3] have shown that indeed, the particle motion in such rapidly oscillating field could be considered as if it occured in an effective *time-independent* potential $V_{eff} = \bigtriangledown V(x) \cdot \bigtriangledown V(x)/4m.\omega^2$, where $V(x)$ is the space-dependent part of the initial potential, m is the particle mass. This result is equivalent to the classical one, since the force is the negative gradient of the potential: $\vec{f} = - \bigtriangledown V(x)$.

Somewhat later author also considered charged particle motion in a rapidly oscillating field in the quantum mechanical framework [4,5]. Contrary to [3], in [4,5] first, particle motion in a *space-periodic high frequency field* was considered and second, there was no any averaging over time. It turns out, that under certain conditions the solution of corresponding Scrödinger equation is asymtotically exact: if the spatial preiod is small enough and field frequency is sufficiently high, the effect of the field on the charged

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particle is in a sense equivalent to the effect of the time-independent effective potential, that is a sequence of deep 'dynamic' potential wells where the electrons are confined. However, in [4,5] only one-dimensional model was considered, which is certainly not up to what could be required, since it is not relevant in full to the real world: in a real world charged particle always has a possibility to escape to other dimensions. Hence, if one would like to treat the results of [4,5] seriously, one should point out physically meaningful situation where such confinement could be observed. So, the aim of this paper is to consider more realistic model of such micro-scale dynamic localisation and to discuss its possible consequences.

2   One-dimensional model

Since the one-dimensional model plays an important role in further considerations, we discuss it here breafly putting the details of calculations into Appendix 1.

So, let us have a charged particle (an electron, for definiteness) moving in a potential of the form $V(x,t) = V(x)\cos\omega t$ where $V(x) = V(x+n\ell) = V_0\cos k_0 x$, and ℓ is the spatial period, n is the integer, $k_0 = 2\pi/\ell$, V_0 is the amplitude. Suppose the movement of electron is governed by the one-dimensional Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = (-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + eV(x)\cos\omega t)\psi, \tag{1}$$

where e,m are the electron charge and mass, repectively. Suppose for a moment that only the potential-energy term were present on the right-hand side of (1). Then the solution of this equation would be

$$\psi(x,t) = \varphi(x,0)\exp(-iV(x)\sin(\omega t)/\hbar\omega).$$

This shows that dominant effect of the potential is to add an oscillating phase factor to the wave function ψ. Hence, as it was suggested in [3], it is natural to look for a solution to Eq (1) of the form

$$\psi(x,t) = \varphi(x,t)\exp(-iV(x)\sin(\omega t)/\hbar\omega). \tag{2}$$

Substituting (2) and $V(x) = V_0\cos k_0 x$ into (1) we have

$$i\hbar\frac{\partial\varphi}{\partial t} = [H_0 + H_1(t) + H_2(t)]\varphi, \tag{3}$$

where

$$H_0 = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + (4m\omega^2)^{-1}(eV_0 k_0 x)^2\sin^2 k_0 x,$$

$$H_1 = -\frac{(eV_0 k_0)^2}{4m\omega^2}\sin^2 k_0 x\cos 2\omega t,$$

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$$H_2 = -\frac{i\hbar}{m\omega}(ek_0V_0 \sin k_0x \frac{\partial}{\partial x} + \frac{1}{2}eV_0k_0^2 \cos k_0x) \sin \omega t.$$

Now it is clear that averaging over time interval much greater than $2\pi/\omega$, or in other words, over the time interval characterising changes of the amplitude $\varphi(x,t)$ and substituting $\sin \omega t$ and $\cos 2\omega t$ by their average values of 0, one would have the particle motion in effective potential

$$V_{eff} = \frac{(eV_0k_0)^2 \sin^2 k_0x}{4m\omega_2} = \frac{\nabla V(x) \nabla V(x)}{4m\omega^2},$$

that is, the result of [3]. But we do not restrict the consideration only to that rough approximation; instead, we do carry out the subsequent analysis in two steps: first, we consider the equation with time-independent right-hand side (i.e., with the Hamiltonian H_0) and then, 'turning on' the terms $H_1 + H_2$, look at the consequences of it. We have then the equation

$$i\hbar\frac{\partial\varphi}{\partial t} = (-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{(eV_0k_0)^2}{4m\omega^2}\sin^2 k_0x)\varphi. \tag{4}$$

Introducing the new variables $\tilde{x} = k_0x$ and $\varphi(\tilde{x},t) = \bar{\varphi}(\tilde{x})\exp(-i\hbar^{-1}\epsilon_nt)$ we obtain

$$\frac{d^2\bar{\varphi}}{d\tilde{x}^2} + (a_n + 2q\cos 2\tilde{x})\bar{\phi} = 0. \tag{5}$$

Here $a_n = -\beta(1 - \varepsilon_n/\alpha)$, $q = \beta/2 = m\alpha/\hbar^2k_0^2$ are dimensionless quantities; $\alpha = (eV_0k_0)^2/8m\omega^2$ and ε_n are the quantities of the dimension of energy.

Equation (5) is the Mathieu equation, the parameter a_n, considered as a function of q, is the eigenvalue related to the corresponding Mathieu function. The Mathieu functions are the eigenfunctions of the Sturm-Liouville problem for the equation (5) and the boundary conditions either

$$\bar{\varphi}(0) = \bar{\varphi}(\pi) = 0, \text{ for } se_n(\tilde{x}, q)$$

or

$$\frac{d\bar{\varphi}(0)}{d\tilde{x}} = \frac{d\bar{\varphi}(\pi)}{d\tilde{x}} = 0, \text{ for } ce_n(\tilde{x}, q) \tag{6}$$

Which of the two conditions and hence, which of the two Mathieu functions, $se_n(\tilde{x}, q)$ or $ce_n(\tilde{x}, q)$ is to be chosen, depends on the particular situation; it'll be discussed a little bit later. Now let us make some estimates. Suppose the characteristic length (spatial period) of the potential $\ell \sim 10^{-4}cm$, $k_0 = 2\pi \times 10^4cm^{-1}$, $\omega \sim 10^{10}Hz$, $V_0 = 1V, q = (1/8)(eV_0/\hbar\omega)^2 = 2.89 \cdot 10^9$. Hence, at so large values of q one can use the asymptote of the eigenvalues [6,7]

$$a_n \sim -2|q| + 2(2+1)|q|^{1/2}; \ n = 0, 1, ...$$

and for the energy values we have

$$\varepsilon_n = (2n + 1)|q|^{-1/2}\alpha. \tag{7}$$

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Note that the obtained 'spectrum' is equidistant with the distance between the levels $\Delta\varepsilon_{n,n-1} = e\hbar V_0 k_0^2 / 4m\omega$, which for the chosen values of paprameters is equal to $\Delta\varepsilon_{n,n-1} = 7.33 \times 10^{-15} erg = 4.5\, meV$.

Let us consider now the solution of (3) with time-dependent $H_1(t) + H_2(t)$ term. In accordance with common quantum-mechanical rules, one should search for the solution to (3) in the form

$$\varphi(x,t) = \sum_n b_n(t)\bar{\varphi}_n \exp(-it\varepsilon_n/\hbar)$$

Here $\bar{\varphi}_n$ are the Mathieu functions from above and the time-dependent coefficients $b_n(t)$ obey the system of equations

$$i\hbar\dot{b}_n(t) = \sum_m H_{nm}(t)b_m(t),$$

where

$$H_{nm}(t) = H_{nm}^{(1)}(t) + H_{nm}^{(2)}(t) = \exp(i\omega_{nm}t)\int_0^{\pi(2\pi)} \bar{\varphi}_n(H_1(\tilde{x},t) + H_2(\tilde{x},t))\bar{\varphi}_m d\tilde{x}.$$

Here $\omega_{nm} = (\varepsilon_n - \varepsilon_m)$.

Depending on whether the functions are π or 2π-periodic, in the last integral the upper limit is π or 2π.

Using the orthogonality and normalization of Mathieu functions, one can prove that $H_{nm}^{(1)} = 0$ if $\bar{\varphi}_n$ and $\bar{\varphi}_m$ are of different parity and $H_{nm}^{(1)} \neq 0$ if $\bar{\varphi}_n$ and $\bar{\varphi}_m$ are of the same parity, that is both are odd or both are even.

Just the opposite, $H_{nm}^{(2)} \neq 0$ only if $\bar{\varphi}_n$ and $\bar{\varphi}_m$ are of different parity and hence, one can treat the terms $H_{nm}^{(1)}$ and $H_{nm}^{(1)}$ independently. It can be shown (see Appendix 1) that the system of equations which the coefficients b_n are to obey, is asymtotically of the form

$$i\hbar\dot{b}_n(t) = \sum_m H_{nm}^{(1)}(t)b_m(t) = \sum_m M_{nm}b_m(t)exp(i\omega_{nm}t). \qquad (8)$$

Here M_{nm} is the real matrix explicit form of which is given in Appendix 1. Using another remarkable asymptotes of the coefficients of corresponding expansions representing Mathieu functions (for details see Appendix 1) and substituting $C_n = b_n \exp(i\lambda_n)$, where λ_n are the parameters to be determined, one has

$$i\hbar\dot{C}_n = \sum_m M_{nm}C_m \exp[it(\omega_{nm} + \lambda_n - \lambda_m)]. \qquad (9)$$

As it is shown in Appendix 1, an appropriate choise of λ_i removes the time dependence from $H_1(t)$, yeilding the solution $\hat{C} = \hat{C}(0)e^{-i\hbar^{-1}tM}$, where $\hat{C}(0) = (C_1(0), C_2(0), ..., C_N(0))$ is the row matrix and N is the maximum number of states (quantum levels) to be included in consideration.

Since the matrix M_{nm} is Hermitian, the solution of (9) finally has the form

$$\hat{C}(t) = \mathrm{diag}[e^{(i/\hbar)\gamma_1 t}, e^{(i/\hbar)\gamma_2 t}, ..., e^{(i/\hbar)\gamma_n t}]\hat{C}(0),$$

where diag[...] is the diagonal matrix, γ_i are the eigenvalues of M, which are not necessary all to be different. So, despite the fact the Hamiltonian in (3) is explicitly time-dependent and the energy, generally speaking, is not conserved, the asymptotic properties of Mathieu function caused by very large value of q and the small ratio of $\omega/\omega_{n,n+1}$ cause the charged particles to be in the states with the energies ε_n and probabilities, corresponding to their initial distribution.

The physical explanation of this consisits of the fact that, since $\omega \ll \omega_{n,n+1}$, the perturbation $H(t)$ can be regarded as adiabatic. But as it is known from quantum mechanics, the adiabatic perturbation cannot cause the transitions between the states of the discrete spectrum [8]. The adiabaticity condition is that the changes of the interaction energy during a period of oscillations in quantum systems are much smaller than the absolute values of the energy differences between the corresponding states [8]:

$$|\omega_{nm}^{-1}\frac{d}{dt} < n|H(t)|m > | \ll \hbar\omega_{nm}$$

So, the existence of the $H(t)$-term in the Hamiltonian results only in the appearance of the new phase factor of the wave function stationary states and hence we can regard the charged particle as moving in some effective time-independent potential $U_{eff} = 2\alpha \sin^2 k_0 x$. Since α, having the dimension of energy, is much greater than the energy of the lowest eigenstates of this potential as well as of $k_B T$, (k_B is the Boltzmann constant) for any reasonable temperature, it is quite obvious that for the electrons this potential is the sequence of a deep 'dynamic' quantum wells where the electrons are confined or localised. However, it is also possible to provide more formal proof of electron localisation in such potential. The formal proof (see Appendix 2) is based on the observation that localisation also means the *mean* electron momentum in such a state should equal to zero.

3   More realistic physical model: quantum wire

Now let us proceed to the real world keeping however in mind and trying to preserve the benifits of one-dimensional model. How could we manage to do that ? Let us undo that 'knot' in a downright fachion, restricting the electron motion in other dimensions. Consider the quantum wire (see Fig.) containing two-dimensional electron gas (2DEG) under quantum Hall-effect conditions. It is known that under these conditions [9] the electron free path in 2DEG can be of the order of 100 μm and even somewhat greater. Suppose the total length L_x of the quantum wire is $L_x \sim \bar{l} = 100\mu m$ and suppose that on the surface of the wire thin insulator layer (its thickness will be estimated a little bit later) is deposited. If the quantum wire would be made of silicon, the insulator layer could be capping oxide. Suppose also that on the surface of the 'sandwich' there are the metal field electrodes in a periodic sequence with the spasing between the electrodes $L_1 \sim 5 \times 10^{-4} cm$. To produce such grating is not a problem for the modern technology;

for example, in [10] the development of a microstructure with a distance between the field electrodes of $100nm$ was reported.

Now by means of suitably patterned gate electrodes, one could impose on a two-dimensional electron gas an artificial periodic potential of the form $U(x, y, t) = (U_1(x) \cdot U_2(y)) \cos \omega t$, where $U_1(x + nL_1) = U_1(x)$. One could also suppose $U_2(y)$ to be weakly dependent of its argument, that is the first and second derivatives of $U_2(y)$ with respect to $y(0 \leq y \leq L_y)$ to be very small. This assumption is quite clear and acceptable ; indeed, the potential applied to the gates is almost uniform along the metal strip that is, within the interval $(0, L_y)$ and changes only at the very edges of the strip in the vicinities of $y = 0$ and $y = L_y$. So, one can conclude that its first and second derivatives are practically equal to zero almost evrywhere within the interval $(0, L_y)$. As for $U_1(x)$, at the moment one cannot say anything escept the artificial potential imposed on 2DEG is a periodic one. But let us investigate more thoroughly the role of the insulator layer.

Since the density of charges in the insulator is much smaller than in semiconductor, it is generally supposed to be zero and hence, one can consider the potential $\Phi(x, y, z)$ within the dielectric as governed by the Laplace equation:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0, \tag{10}$$

If, as we supposed above, the electrodes are suitably patterned and the potential Φ is weakly dependent on y, one can proceed from (10) to the equation

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial z^2} = 0$$

with the boundary condition $\Phi(x, 0, t_0) = U_1(x) \cos \omega t_0 = CU_1(x)$. Since $\Phi(x, 0, t_0)$ is a periodic function with respect to x, $\Phi(x, z)$ could be expanded into a Fourier series of functions $\Phi_n(x, z) = F_n(z) \cos(2\pi x/L_1)$, with the functions $F_n(z)$ obeying the equation

$$\frac{d^2 F_n}{dz^2} = (4\pi^2 n^2/L_1^2)F_n,$$

and hence, $F_n = F_0 \exp(-z/z_n)$, where $z_n = L_1/2\pi n$. Now it is obvious that the amplitude of the n-th harmonic decays exponentialy with the number n inreasing. The natural question is: how thick the insulator layer should be for one could neglect the higher space harmonics, restricting the consideration only to the first one $\sim \cos k_0 x$, where $k_0 = 2\pi/L_1$. If we suppose the amplitude of the second space harmonic is ten times smaller than the first one, we can calculate by means of the formulae above, that the insulator layer thickness should be about $10nm$. This simple analysis shows quite clear that the insulator layer filters off the higher space harmonics of imposed potential and the electrons in the bulk of the semiconductor beneath the insulator are mainly affected by the lowest ones. The higher harmonics are strongly suppressed in the insulator and can be neglected if the insulator is sufficiently thick.

The next question to answer, is this: what the equation governs the electron motion in 2DEG under the circumstances considered above ? The answer may seem to be obvious, but in fact it requires a special care.

It is well-known that the dynamics of electron in the semicnoductor conduction band under the *time-independent* external field can be described by an equation of the form [9]:

$$[E_c + \frac{(i\hbar \bigtriangledown + \mathbf{A})^2}{2m^*} + U(\mathbf{r})]\Psi(\mathbf{r}) = E\Psi(\mathbf{r}),$$

where $U(\mathbf{r})$ is the potential energy due to space-charge etc., $\mathbf{A}$ is the vector potential and m^* is the effective mass. It should be noted that the wave function $\Psi(\mathbf{r})$ we calculate from this equation is not the true wave function but is the smoothed out version that does not show any rapid variations on the atomic scale (see for details [9]).

Look again at the Fig. and suppose at first the constant potential is applied to the gate electrodes. Thus we suppose the electrones are free to propagate in $x - y$ plane and are confined in z-direction. Usually at low temperatures with low carrier densities only the lowest subband corresponding to the confinement is occupied and the higher subbands do not play any significant role. We can then ignore the z-direction altogether and simply treat the semiconductor as a two dimensional system in $x - y$ plane.

By analogy with the last equation, suppose the dynamics of the electrons of lower subband in 2DEG under *time-dependent* external electric field applied to the gate electrodes, is governed by the Schrödinger-like equation of the form:

$$i\hbar\frac{\partial\Psi}{\partial t} = [\frac{\hbar^2}{2m^*}\nabla^2_{x,y} + U(x,y)\cos\omega t]\Psi \tag{11}$$

Here in accordance with the analysis presented above, $U(x,y) = U_1(x)U_2(y)$ where $U_1 = V_0\cos k_0 x$, $k_0 = 2\pi/L_1$ and $U_2(y)$ is supposed to be

$$U_2(y) = \begin{cases} C_1, & 0 < y < L_y \\ C_2, & y = 0, \ y = L_y, \end{cases}$$

where $C_{1(2)}$ are the constants ($|C_1| > |C_2|$). Potenial $U_2(y)$ defined in this way means the jumps of it at the edges of the sample in y-direction, that is the potential discontinuity on the semiconductor-vacuum interface. The last one is equivalent to the Sommerfeld 'rigid box' model of the solid state [11], for which boundary conditions for the electron wave funcrion $\chi(y)$ are: $\chi(0) = \chi(L_y) = 0$.

Searching for the solution to (11) in the form $\Psi(\mathbf{r}) = \chi(y)\varphi(x,t) = \frac{1}{\sqrt{L_y}}\exp(ik_y y)\varphi(x,t)$, one gets the variable separated and together with the boundary conditions, the next system of eqautions holds:

$$-\frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial y^2}\chi(y) = \varepsilon_y\chi(y)$$

$$i\hbar\frac{\partial\varphi}{\partial t} = [\frac{\hbar^2}{2m^*}\frac{\partial^2}{\partial^2 x} + U_1(x)\cos\omega t]\varphi$$

Here

$$\varepsilon_y = \frac{\hbar^2}{2m^*}k_y^2, \ k_y = \frac{2\pi}{L_y}n, n = 0, \pm 1, \pm 2...$$

With $U_1(x) = eV_0 \cos k_0 x$, the second equation becomes the same as that has been considered in previous section. It is clear that in order the approach discussed above to be valid, in particular, for the transformation (2) can be used, the frequency applied to the gate electrodes should be greater than the inverse time which takes for the electrones to fly between two successive gates. The last one can be estimated as follows. Let it be the electron concentration in 2DEG equal $n_s = 5 \cdot 10^{11} cm^{-2}$ and the Fermi velocity is of the order $v_F \sim 3 \times 10^7 cm \cdot s^{-1}$. Then the flight time $\tau = L_1/v_F \approx (5/3) \cdot 10^{-11}s$ and hence, $\tau^{-1} \approx (3/5) \cdot 10^{11} Hz$. It means the condition $\omega \gg \tau^{-1}$ could be fulfilled if ω would be at least of $10^{12} Hz$. However, it is worthy to note that in accordance with the formula for $\Delta\varepsilon_{n,n-1}$, increasing the distance between the gates by a factor 2, one could reduce the friequency of the field applied to the gates by factor 4, retaining the spacing between the levels intact. Thus, if $L_1 \sim 10^{-3} cm$, the frequency can be of $\sim 2.5 \cdot 10^{11} Hz$.

Also we should discuss which of the two boundary conditions (6) for the Mathieu equation has to be chosen. Having in mind possible experimental checking up the proposed model, it seems natural to consider quantum wire connected to the current leads. Then the second of the two conditions (6) is to be more appropriate, since the first one means there is no any current at the quantum wire-lead interfaces.

To complete analysis, make again some estimates. Suppose $L_1 = 10^{-3} cm, \omega \approx 2.5 \cdot 10^{11} Hz, m^* \approx 0.1 \cdot m_e (m_e$ is the free electron mass$), V_0 = 10V$. Then, $q \approx 4.628 \cdot 10^8, \Delta\varepsilon_{n,n-1} \approx 4.5 meV$ and $\Delta\varepsilon_{n,n-1}/\hbar\omega \approx 28$. As a result, we have come to the next conclusion: if in a quantum wire with the periodic gate electrodes considered above and under quantum Hall effect conditions, the spacing is small enough and the frequency of the field applied to the gates is sufficiently high, the effect of the field on the charged paricles is in a sense equivalent to the effect of a time-independent effective potential. The 'potential relief' is a sequence of deep 'dynamic' quantum wells of the depth 2α and the spectrum defined by the formula (7). The physical explanation of this at first sight paradoxical, fact is that, since $\hbar\omega \ll \Delta\varepsilon_{n,n-1}$, the perturbation $H(t)$ can be regarded as adiabatic and hence, it cannot cause the transitions between ε_n- states.

Also it is curious that, despite of the initial potential $U(x) = V_0 \cos k_0 x$ does not look like the harmonic one of the standard quantum mechanical textbooks, the resultant specrtum turns out to be equidistant just like for harmonic potential $\sim x^2$ it is. This fact also can be easily explained. Indeed, at chosen values of the structure parameters, the main dimensionless parameter q which defines all the physics, is of the order of $4.628 \cdot 10^8$ and hence, the effective time-independent potential $U_{eff} = 2\alpha \sin^2 k_0 x$ for small values of x looks like the harmonic one.

4 Discussion and conclusion

In the paper the dynamic localisation of charged particle (electron) in a quantum wire under the external non-uniform time-dependent field is considered. The electrons are trapped in a deep 'dynamic' quantum wells (stricktly speaking, quantum dots, since the electron 'energy spectrum' turns out to be completely discrete in there) which are the result of specific features of the potential imposed on 2D electron gas: the scale of spatial nonuniformity is much smaller then the electron mean free path ($L_1 \ll \bar{l}$) and the frequency is much greater then τ^{-1}, where τ is the electron free flight time. As a result, the effect of this field on the charged particle is in a sense equivalent to the effect of a time-independent effective potential, that is a sequence of deep 'dynamic' quantum wells were the elelctrons are confined.

Certainly the next question immediately arises. Whatever large, the dimensionless parameter q is not however, *infinitely* large. But mathematically it is just not we need: we should have $q \to \infty$, in order to neglect some part of the perturbation. So, what are the consequencies of the q is finite ?

It is quite clear that the picture sketched above is just an approximation. Since q though large, is finite, neglected part of the perturbation would cause the transitions between the levels. However, the rate of the transitions is small compared to the inverse field frequency. There are should be about 20 or 30 of field oscillations for the transition probability reaches the unit. The complete analysis of charge carriers transport in the 'dynamic' quantum wells under such transitions is beyond the scope of the paper; we plan to discuss it in the next publication. Here we note only that the localisation considered above seems to lead on one hand, to the current suppression in a quantum wire and on the other, to the emerging of current peaks on its current-voltage characteristic (so called, *I-V* curve). These peaks are due to electron delocalisation, or escaping from the wells by means of resonant tunnelling which seems to be quite possible, since the tunnelling time (do not confuse it with the tunnelling rate which is proportional to the inverse tunnelling probability) is much smaller then the rate of transtions between the ε_n-states. Indeed, omitting the subtle question of prooper definition of tunnelling time (see discussion on that subject in [12]), we could estimate it roughly as $\tau_{tl} \sim s_{br}/v_g$, where s_{br} is the width of the 'barrier' between two successive dynamic wells and v_g - group velocity which could be supposed equal to Fermi velocity. Supposing s_{br} to be approximately ten times smaller than L_1 (which seems quite reasonable), we get then τ_{tl} 200 or even 300 times smaller then the transmission rates. Hence we conclude that for tunnelling electrons the 'potential relief' they encounter on their way indeed looks like the static one.

Putting it into plain words, one can say that since the electron life time in ε_n- states is long enough (the transition rates as we remember, slow down), no wonder that just for these states probability of tunnelling increses.

Suppose we have a sequence of barriers on which partciles are incident and suppose the energy of particles is smaller than the height of barriers. Then the barriers are

practically impenetrable for almost all particle energies; however, for certain discrete energies and respective energy level widths particle can pass through the barriers without any reflection. This is the resonant tunnelling phenomenon [13,14]. It is quite remarkable that from the mathematical point of view the transparency of the barriers for these discrete energies does not depend on the width of the barriers if the barriers are identical [13].

Return now to the stationary equation (4) and rewrite it in the form:

$$\frac{\hbar^2}{2m^*}\psi'' + (\epsilon - U_{eff}(x))\psi, \tag{12}$$

where

$$U_{eff}(x) = \begin{cases} 2\alpha \sin^2 k_0 x, & x \in [0, L_x] \\ 0, & x < 0, \ x > L_x. \end{cases}$$

The tunnelling problem consists of finding all the solutions of (12) parametrically dependent on $\epsilon \in (0, \infty)$ and behaving as

$$\psi = \begin{cases} \exp(ipx) + r(p)\exp(-ipx) & x < 0, \\ t(p)\exp(ipx) & x > L_x; \end{cases}$$

(the case of a normalized particle beam incident from the left); $p^2 = \epsilon$; $t(p)$, $r(p)$ stand for transition and refelection amplitudes, respectively.

We introduce now the outgoing propagator $G^+(x, x'; p)$ and define the electron wave function along the internal region $0 \le x \le L_x$ by $\psi(p, x) = 2ipG^+(0, x; p)$. Then the transmission amplitude is $t(p) = 2ipG^+(0, L_x; p)\exp(-iL_x)$. Near an isolated pole $p_n = \delta_n - i\gamma_n$ it is possible to write the propagator in the form [13]:

$$G^+(x, x'; p) \approx \frac{\bar{\varphi}_n(x)\bar{\varphi}_n(x')}{2p(p - p_n)}. \tag{13}$$

Here $\bar{\varphi}_n(x)$ are the eigenfunctions of the Hamiltonian in the left-hand side of (12), $p_n^2 = \varepsilon'_n - i\Gamma_n/2, \varepsilon'_n = \delta_n^2 - \gamma_n^2, \Gamma_n = 4\delta_n\gamma_n$. Using (13) and taking into account that $\delta_n \gg \gamma_n$, for the transmission coefficeints $|t(p)|^2$ we have

$$|t(p)|^2 = \frac{p^2|\bar{\varphi}_n(0)|^2|\bar{\varphi}_n(L_x)|^2}{\delta_n^2[(p - \delta_n)^2 + \gamma_n^2]}.$$

Let us notice that in accordance with [13], $\gamma_n = (|\bar{\varphi}_n(0)|^2 + |\bar{\varphi}_n(L_x)|^2)/2I$, where $I \approx 1$. It is also clear from the formula for $|t(p)|^2$ why we choose the second of the two boundary conditions (6).

If the potential in (12) possesses only two maxima (consists of two barriers), i.e. $L_x = 2\pi/k_0$, then, since the barriers are symmetric, i.e. $|\bar{\varphi}_n(0)|^2 = |\bar{\varphi}_n(L_x)|^2$, we have

$$\lim_{p \to \delta_n(\epsilon \to \varepsilon'_n)} |t(p)| = 1$$

Now, let the number of identical barriers be $m > 2$; then as it was proved in [14], if any arbitrary pair of barriers is completely transparent at some energy $\epsilon = \varepsilon_n$, the whole structure consisted of $m > 2$ replicas is also completely transparent at this energy.

Thus it may be said that just like in the static quantum wells, in the sequence of 'dynamic' ones, the electron resonant tunnelling also seems possible because the barrier transmition coefficient has maxima if the electron energy $\epsilon \rightarrow \varepsilon_n$.

To our mind, such 'dynamic' quantum wells would have even some advantages in comparison with static semiconductor heterostructures, since their 'spectrum' could be controlled by the external high-frequency electric field applied to the gate electrodes.

In previous section we estimated the field frequecny ω as to be equal $\sim 2.5 \times 10^{11} Hz$. This value is determined mainly by the electron mean free path which under quantum Hall effect conditions can be about $100 \mu m$ and hardly could be greater. To apply the field of such frequecny to the microstructure, whatever possible, is difficult problem to solve and it is interesting to know whether it is possible to reduce the frequency. We plan to consider this really intriguing problem in the next publication, here instead, we discuss another interesting question: how the effect considerd in the paper relates to the quantization of charge carrier energy spectrum in the *uniform* electric field, that is to Wannier-Stark effect.

In 1960, G.H. Wannier [15] studied electronic states in the presence of a uniform electric field and found that eigenstates are localised along the direction of the electric field and have quantized energy levels $\epsilon_n = neEd$, where n is an integer, E the electric field and d the lattice period along the electric field.

The charge carriers in a crystal in the external electric field, in order to pass from the domain of the lower potential to that one of the higher potential, should gain an additional energy. Hence, the electrons in a crystal in an external electric field are to be localised in a domain with the characteristic length of the order of $2\Delta_s/eE$, where Δ_s is the subband width. As a result, the electron energy spectrum in a pure crystal whithout impurities would resemble the ladder which is called Stark, or Wannier-Stark ladder. Therefore, the electron dynamics along the field direction, say x-direction, can be characterised by the k_x-momentum only for those values of E, for which localisation length are much greater than d. This field range, $2\Delta_s \gg eEd$ is called classical. If the field increases, the situation may come into being when the localisation length becomes of the order of d. It is clear that the energy dependence on the qausi-momemtum k_x is no longer parabolic, the electron turns out to be trapped in the potential well and its dynamics practically does not depend any further on the crystal lattice parameters. The corresponding field range, $2\Delta_s \geq eEd$ is called quantum.

The localised Stark ladder states are associated with Bloch oscillations [16]. The frequency of Bloch oscillations is $\omega_B = eEd/\hbar$. In order to observe the Bloch oscillations, one should have the next condition fulfilled: $\omega_B \tau_p \geq 1$, where τ_p is the momentum relaxation time. Since usually in the 3D semiconductors τ_p is of the order of $10^{-13}s$, the field should be about $2 \cdot 10^5 V \dot{c} m^{-1}$ in order to satisfy this condition. Usually even the lower fields cause the electric breakdown and that is why the Bloch oscillations were not

observed in bulk semicnoductors.

There have been also a number of other controversial arguments on the Stark ladder states. Some claimed [17] that such ladder states cannot exist because they will decay rapidly due to Zener tunnelling caused by interband mixing. Some others claimed that such interband effects are totally absent and no Zener tunnelling is observable [18]. Others claimed that Zener tunnelling is possible in Stark ladders [19,20].

A number of experiments has been done in order to observe a Stark ladder in bulk materials, but no clear evidense was found. Only weak oscillations of conductivity observed in ZnS was attributed to the hopping motion of electrons between Stark ladder states [21]. The failure of observation is attributed to scattering by impurities or phonons, which prevent the accelaration of electrons up to a Brillouin-zone edge. However, in superlattices electrons can be accelerated easily to the edge of the Brillouin zone before they are scattered because of the large lattice constant. Since then, a number of experiments on Wannier-Stark effects have been performed using various techniques (see [22]).

It is clear that the effect discussed in the paper though different, resembles to some extent Wannier-Stark localisation , because it is the electron localisation in *high-frequency non-uniform* electric field. Since the localisation length is much greater than d and the momentum relaxation time in quantum wires under quantum Hall effect conditions is much greater than in bulk semiconductor, one can hope that these states could be observable just like Wannier-Stark states are observable in the superlattices.

For example, it is known that magnetophonon resonance (MPR) is observable in the superlattices [23]. The MPR is due to the oscillating behaviour of the electron density of states arising as a result of the Landau quatization. The magnetophonon resonance appears every time when the phonon frequency ω_{LO} is equal to the cyclotron frequency ω_c in a magnetic field multiplied by small integer n, $\omega_{LO} = n\omega_c, n = 1, 2, ..$ where $\omega_c = eB/m^*$. Thus, it seems probable, that in case of the localisation effect discussed here, the current peaks on the $I - V$ characteristics of the structure also could be observed when the optical phonon frequency ω_{LO} would be equal to $\Delta\varepsilon_{nm}/\hbar$. Here ε_n-states would play the same role as the Landau levels do in MPR and the last phenomenon, if observed, could be called *electrophonon* resonance in the high-frequency non-uniform electric field.

As the final remark, it is worthwhile to return again to the beginning of the paper in order to compare the effect discussed here with the Paul trapping effect and emphasize their striking similarity.

Indeed, the ion motion in radio-frequency non-uniform electric field to the first approximation can be represented as the sum of comparatively slow motion $\mathbf{r}(t)$ in some effective potential and the rapid oscillations with small amplitude $\eta(t)$ near the local equilibrium $\mathbf{r}_0(t)$:

$$\mathbf{r}(t) = \mathbf{r}_0(t) + \eta(\mathbf{r}_0)\cos\Omega_0 t,$$

where Ω_0 is the applied field frequency and the amplitude of small oscillations $\eta(t)$ is

determined by the trap electric field:

$$\eta(\mathbf{r}_0) = eE(\mathbf{r}_0)/M\Omega_0^2,$$

here M is the ion mass and the effective potential is of the form [24,25]:

$$\Phi_{eff} = \frac{eA^2}{M\Omega_2^2}(\rho_0^2 + 4z_0^2).$$

Parameter A is defined as $A = U_0/(\rho_0^2 + 2z_0^2)$, U_0 is the amplitude of the field and ρ_0, z_0 are the cylindrical coordinates of $\mathbf{r}_0$. In this effective potential an ion oscillates along z-axis with the frequency $\bar{\omega}$ and in the $x - y$ plane with the frequency $\omega_\rho = \bar{\omega}/2$, where

$$\bar{\omega} = 2\sqrt{2}eA/M\Omega_0.$$

More thorough analysis [25] has shown that the oscillations in the effective potential Φ_{eff} are stable if the parameter $\Omega_0/\bar{\omega}$ is sufficiently large. This analysis also showed the ion oscillation spectrum consists of the *infinite set* of discrete frequencies $n\Omega_0 \pm \bar{\omega}, n = 0, \pm 1, \pm 2....$

It is easily seen that our condition $\omega \gg \tau^{-1}$ corresonds to the stability parameter introduced in [25] and the states ε_n correspond to the spectrum of ion oscillations in the Paul trap.

5 Acknowledgements

6 Appendix 1

Here in Appendix 1 we derive the system of equations (8) to which the b_n-coefficients are obeyed. Let the Mathieu functions $\bar{\varphi}_n^0$ and $\bar{\varphi}_m^0$ be of the form (see[6])

$$\bar{\varphi}_n = ce_{2n}(\tilde{x}, -q) = (-1)^n \sum_{r=0}^{\infty} (-1)^r A_{2r}^{(2n)} \cos 2r\tilde{x},$$

$$\bar{\varphi}_m = ce_{2n}(\tilde{x}, -q) = (-1)^n \sum_{r=0}^{\infty} (-1)^r A_{2r}^{(2m)} \cos 2r\tilde{x}.$$

Using the orthogonality of the functions one can prove that $H_{nm}^{(1)} = \alpha f_{nm} \cos 2\omega t$, where

$$f_{nm} = \frac{1}{2} \sum_{r=0}^{\infty} \sum_{s=o}^{\infty} (-1)^{r+s} A_{2r}^{(2n)} A_{2s}^{(2m)} (\delta_{r-1,s} + \delta_{r+1,s})$$

$$= -\frac{1}{2} (A_2^{(2n)} A_0^{(2m)} + A_4^{(2n)} A_2^{(2m)} + ... + A_0^{(2n)} A_2^{(2m)} + A_2^{(2n)} A_4^{(2m)} + ...).$$

The evaluation of the sum in the brackets requires special care, but one can note that the asymptotic behaviour of $A_{2r}^{(2n)}$ at $q \to \infty$ does not depend on the superscript. For example (see [6]): $\lim_{q\to\infty}(A_2^{(2n)}/A_0^{(2n)}) = -2$, $\lim_{q\to\infty}(A_{2r}^{(2n)}/A_0^{(2n)}) = (-1)^r 2$ at arbitrary n.

Using the recurrence formula for the function ce_{2n}

$$a_{2n} A_0^{(2n)} - q A_0^{(2n)} = 0,$$

one can demonstrate that for $q = 1600$ the ratio $A_2/A_0 = -1.95$ for the function ce_0 and -1.75 for the function ce_2. At the $q = 4.628 \cdot 10^8$ we have $A_2/A_0 = -1.999907$ for ce_0 and -1.99972 for ce_2, respectively. Hence we conclude that

$$f_{nm} \approx - \sum_s A_{2s}^{(2n)} A_{2(s-1)}^{(2m)} \tag{14}$$

and

$$H_{nm}^{(1)} = \alpha f_{nm} \cos 2\omega t \exp(i\omega_{nm} t) = \frac{1}{2} \alpha f_{nm} (\exp i(\omega_{nm} + 2\omega)t + \exp i(\omega_{nm} - 2\omega)t).$$

Note that at the chosen values of parameters $2\omega \ll \omega_{nm}$, and hence
$$H_{nm}^{(1)} \approx M_{nm} \exp(i\omega_{nm}t), \quad M_{nm} = \alpha f_{nm}/2.$$

Suppose now that
$$\bar{\varphi}_n = ce_{2n+1}(\tilde{x}, -q) = (-1)^n \sum_{r=0}^{\infty} (-1)^r B_{2r+1}^{(2n+1)} \cos(2r+1)\tilde{x},$$
$$\bar{\varphi}_m = ce_{2m}(\tilde{x}, -q).$$

Then $H_{nm}^{(2)} = \mu g_{nm} \sin \omega t$, where $\mu = -i2\hbar ek_0^2(V_0/m\omega)$ and
$$g_{nm} = \sum_{r=1}^{\infty} r A_{2r}^{(2m)} (B_{2r+1}^{(2n+1)} + B_{2r+3}^{(2n+1)}).$$

The last series is convergent, since coefficients $A_{2r}^{(2m)}, B_{2r+k}^{(2n+1)}$ are of the order of r^{-2} at $r \to \infty$. It is noteworthy that the coefficients $B_{2r+1}^{(2n+1)}$ also do not depend on the superscript at $q \to \infty$: $\lim_{q\to\infty} B_{2r+1}^{(2n+1)}/B_1^{(2n+1)} = (-1)^r$. So, $H_{nm}^{(2)}$ has the form
$$H_{nm}^{(2)} = \frac{\mu g_{nm}}{2i}(\exp i(\omega_{nm} + \omega)t - \exp i(\omega_{nm} - \omega)t)$$

and for $q \to \infty$ and $\omega/\omega_{n,n+1} \ll 1$, $H_{nm}^{(2)} \to 0$. Finally we have
$$i\hbar\dot{b}_n(t) = \sum_m H_{nm}^{(1)}(t)b_m(t) = \sum M_{nm} \exp(i\omega_{nm}t)b_m(t), \qquad (15)$$

that is, Eq (8). Owing to the absolute convergence of the series (14), one can conclude that f_{nm} does not exceed some finite quantity. Also it is known [6] that $A_{2s}^{(2n)}$-coefficients possess another remarkable asymptote, namely $A_{2s}^{(2n)} \to 0$ for $n \to \infty$ and $n > q$. Hence, we can restrict the number of states to consider by a finite number N. Substituting $C_n = b_n \exp(i\lambda_n)$ where λ_n are the parameters to be determined, one gets
$$i\hbar\dot{C}_n = \sum_m M_{nm} \exp it(\omega_{nm} + \lambda_n - \lambda_m)C_m.$$

Thus, any choice of λ_i such that for all n, m
$$\omega_{nm} + \lambda_n - \lambda_m = 0 \qquad (16)$$

removes the time dependence from $H_1(t)$, yelding the solution $\hat{C} = \hat{C}(0) \exp(-i\hbar^{-1}Mt)$, where $\hat{C} = (C_1(0), C_2(0), ..., C_N(0))$ is the row matrix consisting of coefficients at the initial time $t = 0$.

It is obvious that in general (16) includes up to $(1/2)N(N-1)$ equations in the N unknowns λ_i to be solved simultaneously, so that the consistent choice of the set $\{\lambda_i\}$ is not always possible. However, we have to recollect that our 'spectrum' ε_n is equidistant and in case of N states the set $\{\omega_{nm}\}$ consists of $N-1$ unequal terms only. So, if we believe any of the λ_i is equal to some definite value (zero, for example) we obtain $N-1$ simultaneous equations in $N-1$ unknowns. Thus, as it is stated in Sec.1, we have
$$\hat{C}(t) = \mathrm{diag}[e^{(i/\hbar)\gamma_1 t}, e^{(i/\hbar)\gamma_2 t}, ..., e^{(i/\hbar)\gamma_n t}]\hat{C}(0),$$

where diag[...] is the diagonal matrix and γ_n are the eigenvalues of M-matrix .

7 Appendix 2

Here we prove the *mean* electron momentum is equal to zero in the potential $U_{eff} = 2\alpha \sin^2 k_0 x$, using the the Wigner function method [26].

Soppose the small subsystem described by the Hamiltonian

$$H = -\frac{\hbar^2}{2m^*}\frac{d^2}{dx^2} + 2\alpha \sin^2 k_0 x$$

is in the state of thermal equilibrium with the environment characterised by the temperature T. Thus, the density matrix of the subsystem is of the form [8]:

$$\rho(x, x') = Z^{-1} \sum_n \bar{\varphi}_n^*(x)\bar{\varphi}_n(x') \exp(-\varepsilon_n/k_b T),$$

where statistical sum

$$Z = \sum_n \exp(-\varepsilon_n/k_B T) = Sp[(-H/k_B T)]$$

allows the density matrix to obey the normalization condition $Sp[\rho] = 1$.

The diagonal element $\rho(x, x) = P(x)$ is a probabilty for a particle to be at the point with the coordinate x, while $\rho(p, p) = P(p)$ is a probablility for a particle to have the moment p.

In the framework of classical statistical mechanics there can be introduced probability density function with the properties:

$$P(p) = \int f(p, x) dx, \ \ P(x) = \int f(p, x)\frac{dp}{2\pi\hbar}.$$

It turns out that in quantum mechanics the role of probability density function plays Wigner function $f_W(p, x)$ defined as [26]:

$$f_W(p, x) = \int \rho(x + \eta/2, x - \eta/2) \exp((-i/\hbar)p\eta) d\eta. \tag{17}$$

Then, for a function $h(p, x)$ which is the function only of p, or only of x, the next relation holds:

$$< h(p, x) > = \int f_W(p, x)h(p, x)\frac{dp}{2\pi\hbar} dx,$$

where $< .. >$ stands for mean value. Thus, in order to calculate $< p >$, we need in accordance with (17), to have the expression for the density matrix $\rho(x, x)$. With Hamiltonian considered, the equation for density matrix reads

$$i\hbar\frac{\partial}{\partial t}\rho(x, x') = (H - H')\rho(x, x'; t). \tag{18}$$

Since the Hamiltonian is time-independent, one can search for the stationary solution to this equation. It is easily seen, that if one searches for the solution in the form

$$\rho(x, x') = C \exp(-f(x + x', x - x')),$$

it is possible, by the proper choise of the auxiliary function $f(x + x', x - x')$ to subtract the potential U_{eff}, sutisfying in this way the equation (18).

One can check it up that the proper choise is this:

$$\rho(x, x') = C \exp[-\frac{m^*}{a}(\frac{1}{2\hbar k_0})^2 \sin(k_0(x - x')) - 2a\alpha \cos(k_0(x + x'))], \qquad (19)$$

where a is the arbitrary constant and C is the normalizing constant defined by the usual condition

$$\int_0^{L_x} \rho(x, x')dx = 1.$$

Supposing $a = i\mu$, upon expanding the exponential of the trigonometric function in terms of Bessel functions [27]:

$$\exp(iy sin\vartheta) = \sum_{n=-\infty}^{\infty} J_n(y)e^{in\vartheta},$$

where $y = (m^*/\mu)(1/2\hbar k_0)^2$, $\vartheta = k_0\eta$ and using (17), (19), by means of direct calculation one arrives at $< p >= 0$.

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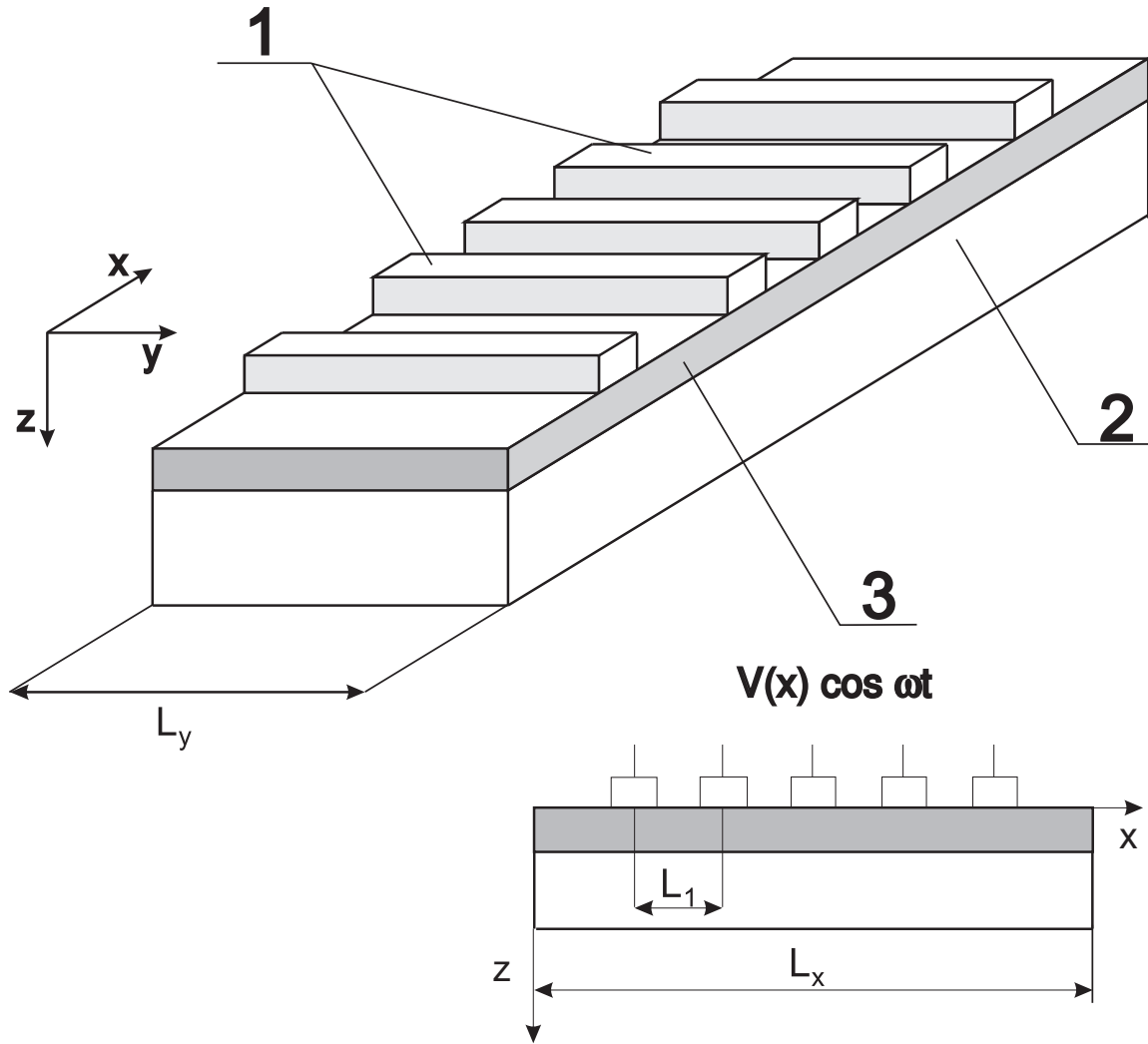
1

x

y

z

L_y

2

3

$V(x) \cos \omega t$

x

L_1

z

L_x

Figure Captions

A sketch of the quantum wire with periodic gate electrodes. 1: metal gates; 2: insulator layer; 3: semiconductor with 2DEG under quantum Hall effect conditions.